Lezione 4

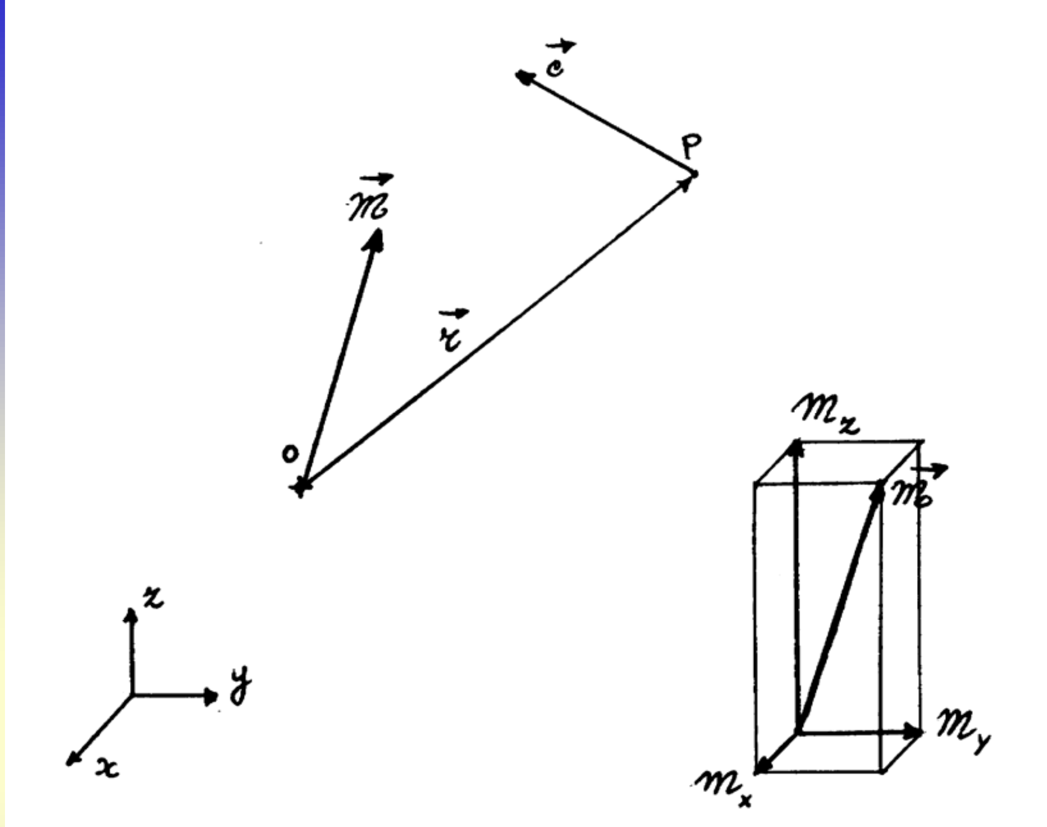
Equazione di Eulero per le turbomacchine

2° principio della dinamica

- Teorema dell’impulso e della quantità di moto
- Macchine rotanti => momento dell’impulso e momento della quantità di moto

Sistemi discreti

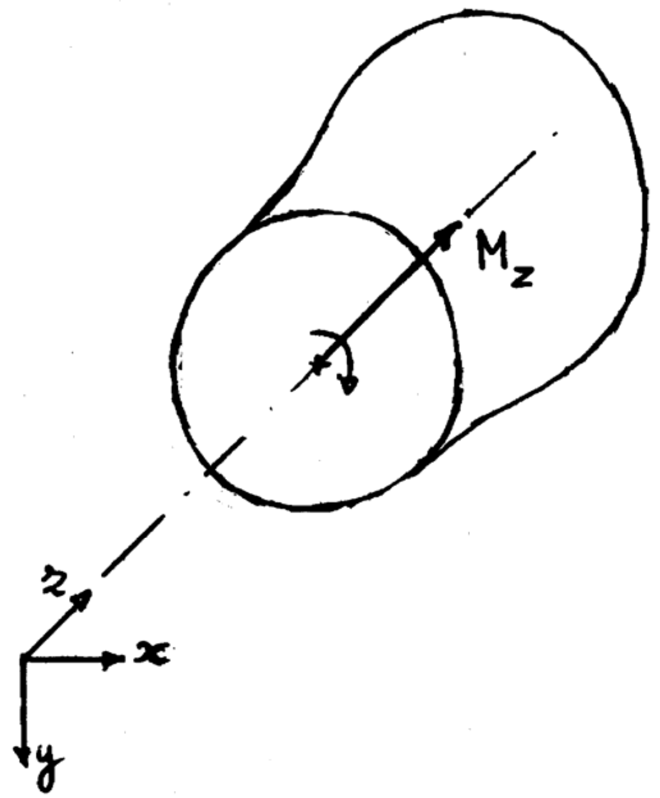
$$\Delta\vec{\mathfrak{M}} = \left(\sum_j \vec{M}_j\right)\Delta t$$



$$\vec{\mathfrak{M}} \equiv \vec{r} \wedge m\frac{d\vec{r}}{dt} = \vec{r} \wedge m\vec{c}$$

Sistemi continui

$$\vec{\mathcal{M}} = \int_V \vec{r} \wedge \rho \vec{c} \, dV$$

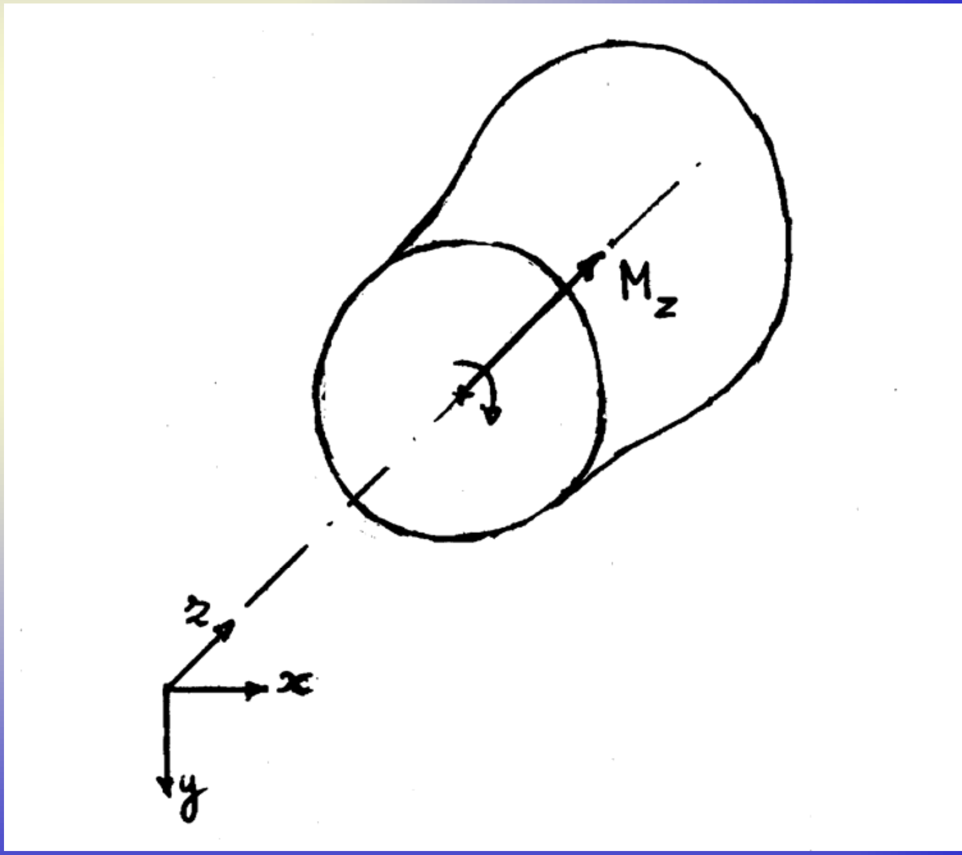


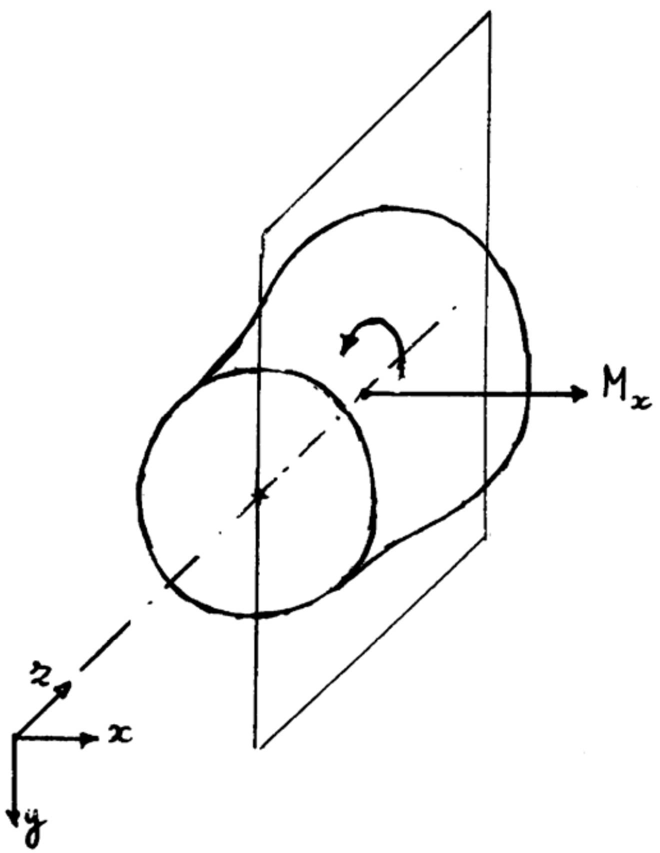
Metodo lagrangiano

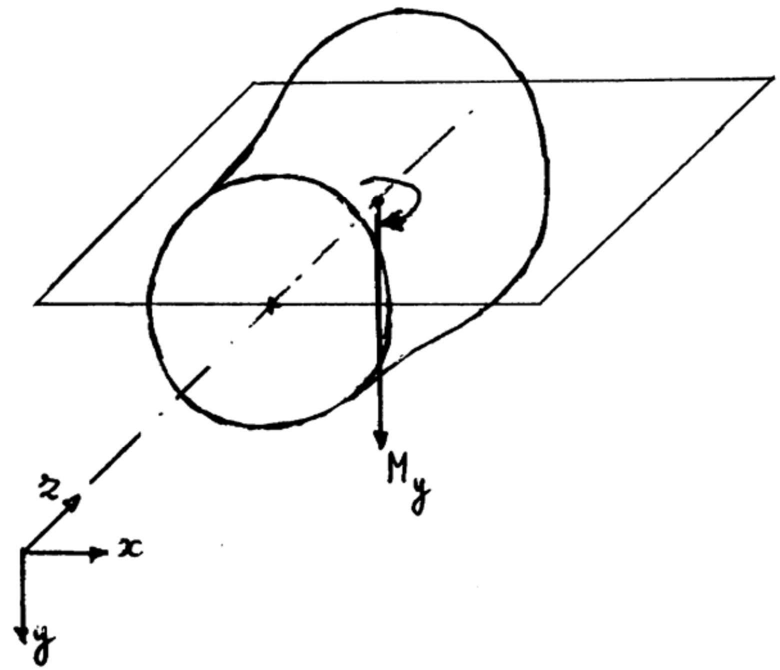
$$\begin{cases} d\mathcal{M}_x = \left(\sum_j M_{jx} \right) dt \\ d\mathcal{M}_y = \left(\sum_j M_{jy} \right) dt \\ d\mathcal{M}_z = \left(\sum_j M_{jz} \right) dt \end{cases}$$

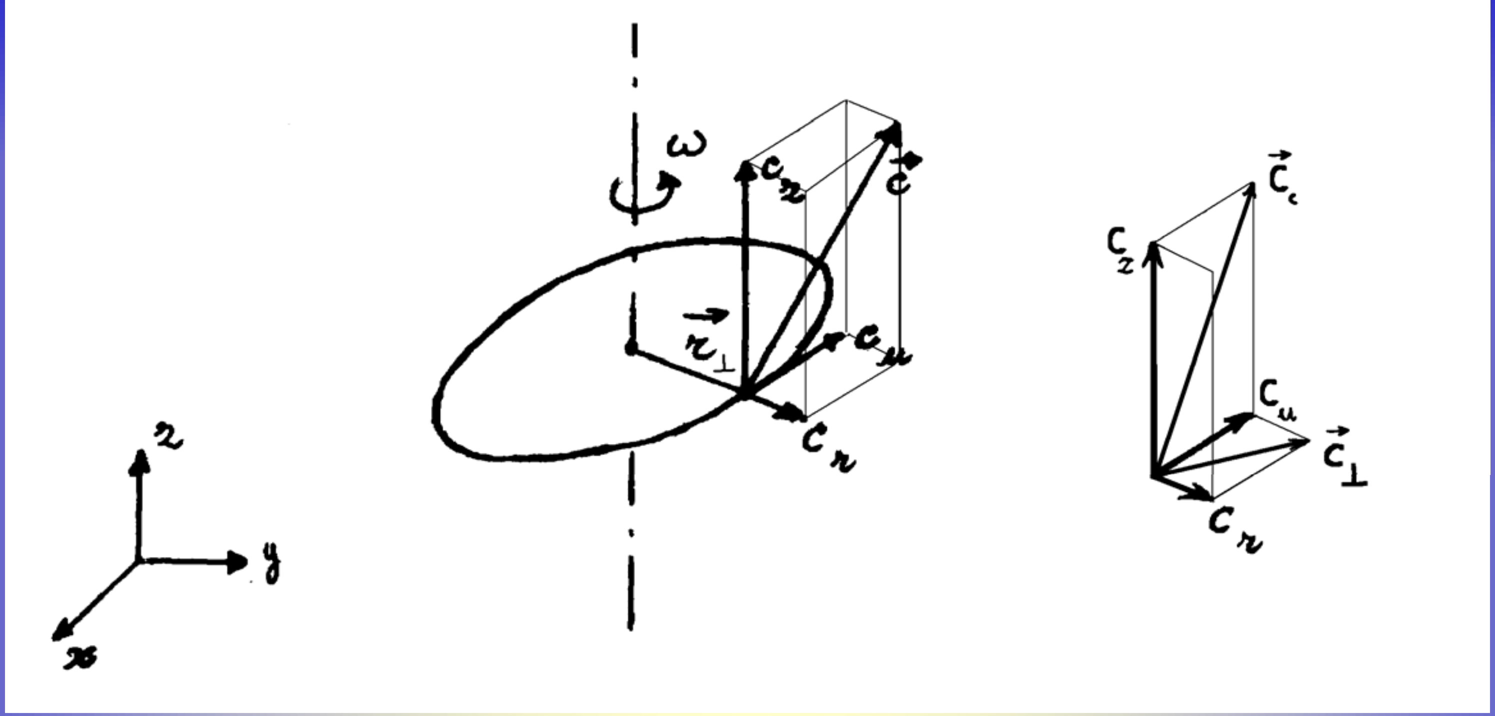
$$d\hat{L}' = \left(\vec{M}_M \bullet \vec{\omega}\right)dt = \left(M_{Mx}\omega_x + M_{My}\omega_y + M_{Mz}\omega_z\right)dt$$

$$= M_{Mz}\omega_z \, dt$$




Mx
z
x
y


My
z
x
y

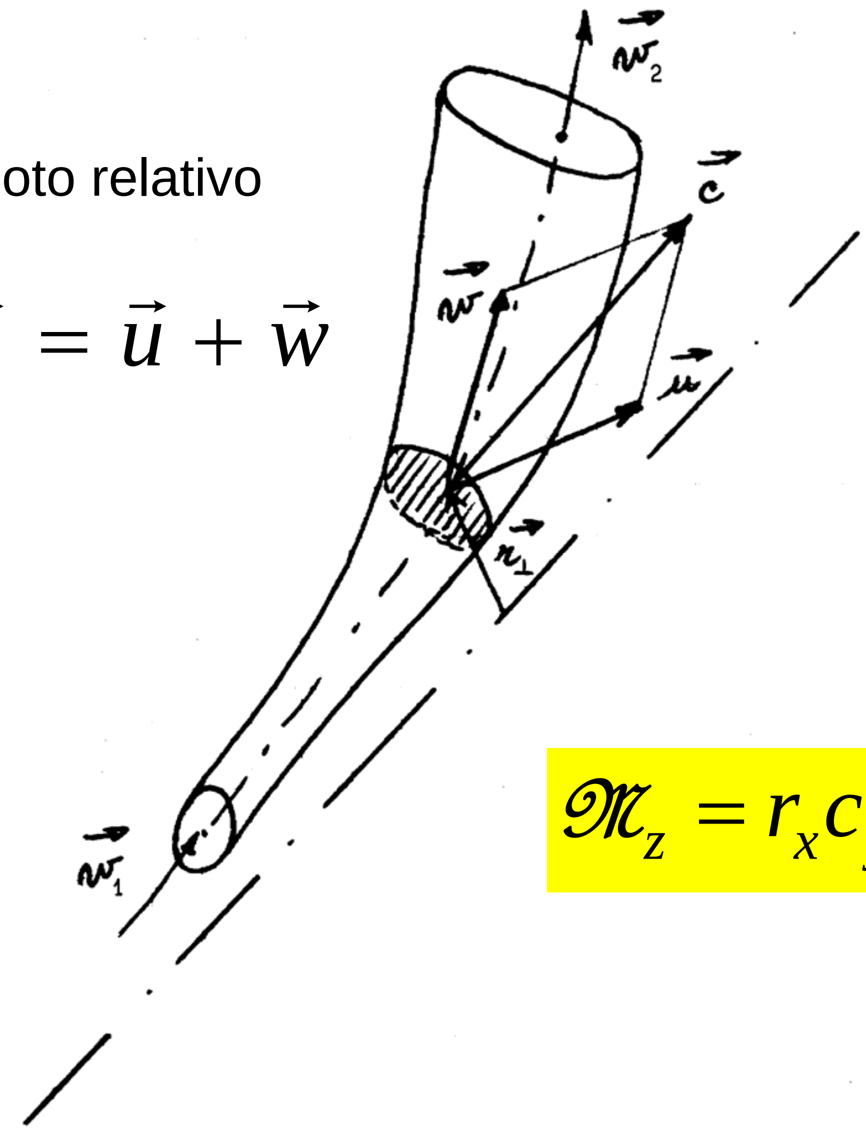


$$\vec{\mathcal{M}} = \vec{r} \wedge \vec{c} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ r_x & r_y & r_z \\ c_x & c_y & c_z \end{vmatrix} = \left(r_x c_y - r_y c_x\right)\vec{k} + \left(r_z c_x - r_x c_z\right)\vec{j} + \left(r_y c_z - r_z c_y\right)\vec{i}$$

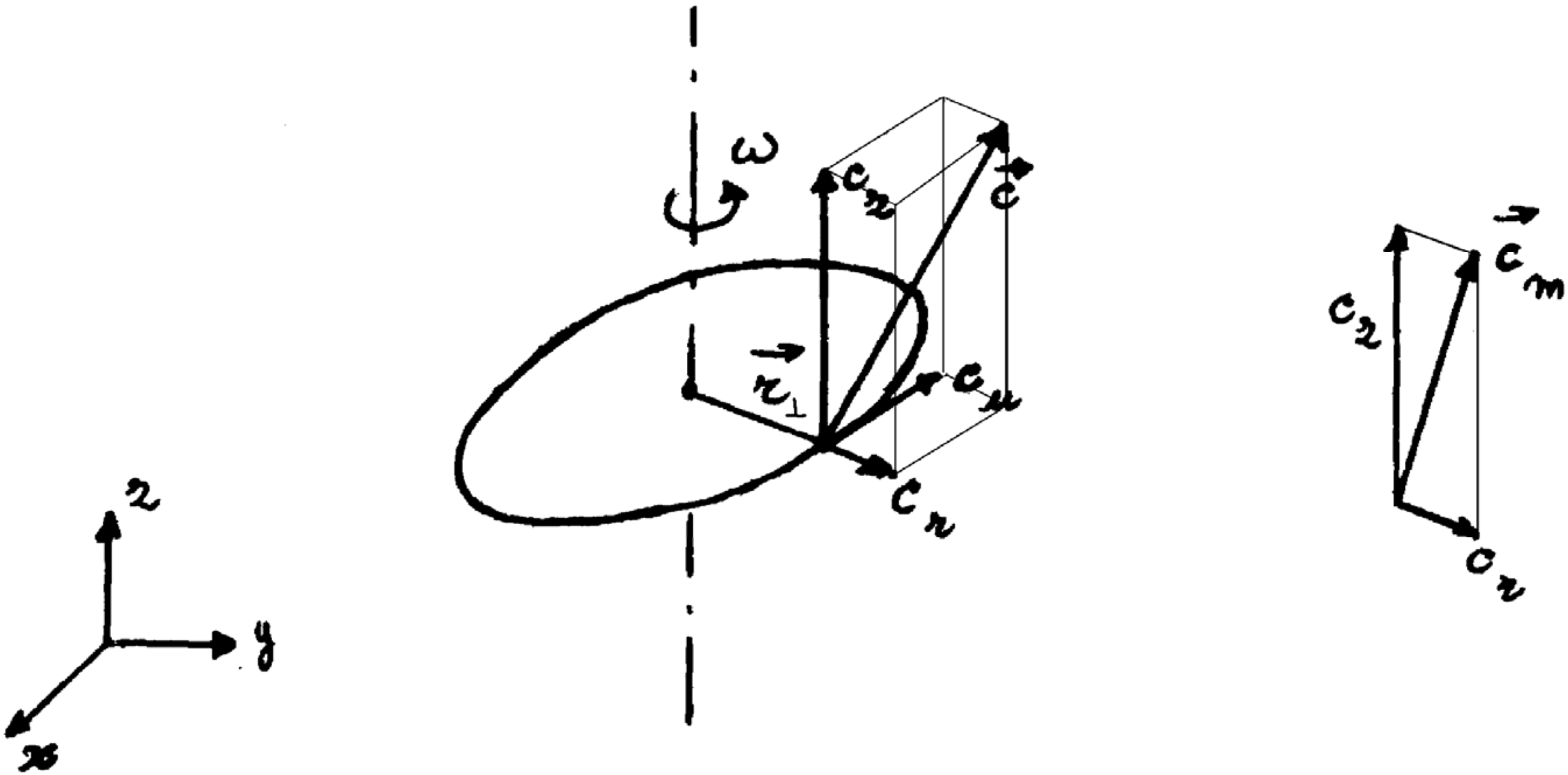
$$\mathcal{M}_z = r_x c_y - r_y c_x$$

Moto relativo

$$\vec{c} = \vec{u} + \vec{w}$$



$$\mathcal{M}_z = r_x c_y - r_y c_x = r_\perp c_u$$



ω
c z
c
r ⊥
c u
c r
c m
c z
c r
z
y
x

Ipotesi di Eulero

- Moto modimensionale (1D)
- Stazionario
- Assialsimmetrico

Equazione dell'impulso e del momento angolari

$$\left\{\begin{array}{c}\text{momento della}\\ \text{q.di moto che entra}\\ \text{nel sistema attraverso}\\ \text{la sup. permeabile}\\ \text{nel tempo } dt\end{array}\right\} - \left\{\begin{array}{c}\text{momento della}\\ \text{q.di moto che esce}\\ \text{dal sistema attraverso}\\ \text{la sup. permeabile}\\ \text{nel tempo } dt\end{array}\right\} + \left\{\begin{array}{c}\text{momento ris.}\\ \text{delle forze}\\ \text{esterne agenti}\\ \text{sul sistema}\end{array}\right\} dt = \left\{\begin{array}{c}\text{accumulo di}\\ \text{momento angolare}\\ \text{nel sistema durante il}\\ \text{tempo } \quad dt\end{array}\right\}$$

$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow \qquad (97)$$

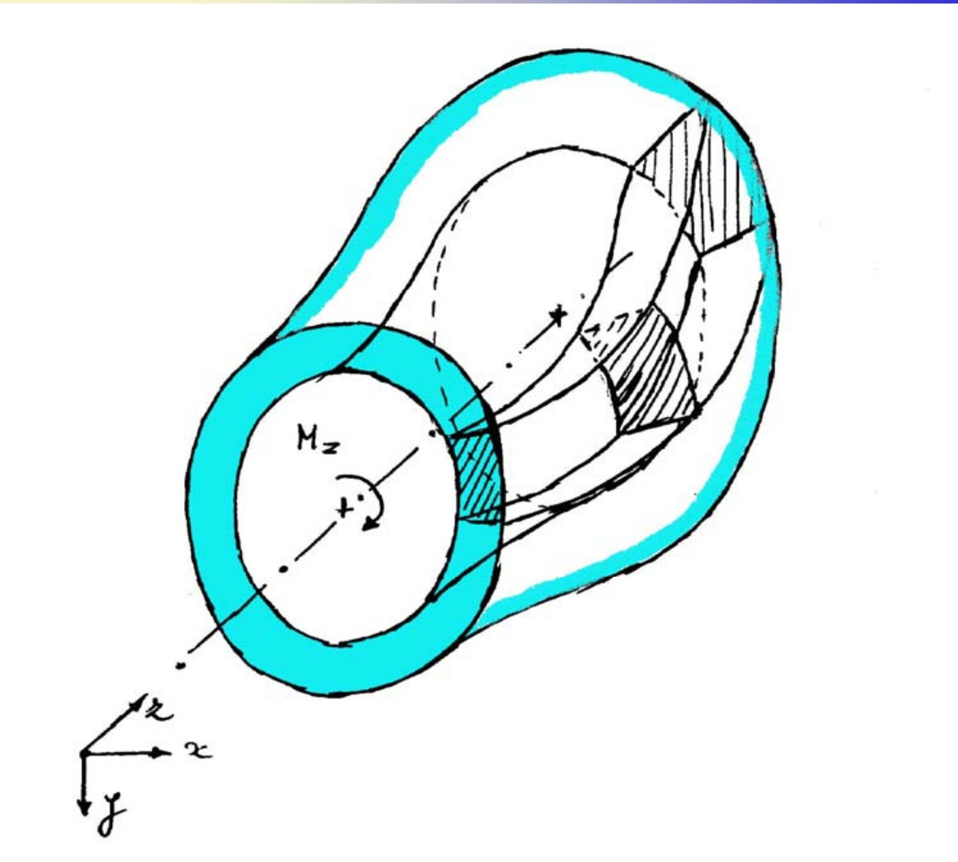
$$\dot{\mathcal{M}}_{ez}\, dt \quad - \quad \dot{\mathcal{M}}_{uz}\, dt \quad + \left[M_{Vz} - M_{Az} + M_{\Omega z} \right] dt = \frac{\partial}{\partial t}\left(\int_V \rho \left| \vec{r}_\perp \right| c_u \, dV \right) dt$$

$$\vec{\dot{\mathcal{M}}} = \int_\Omega \vec{r} \wedge \rho \vec{c} \left(\vec{c} \bullet \vec{n} \right) d\Omega$$

$$\dot{\mathscr{M}}_z = \int_\Omega r \rho c_u \left(\vec{c} \bullet \vec{n}\right) d\Omega$$

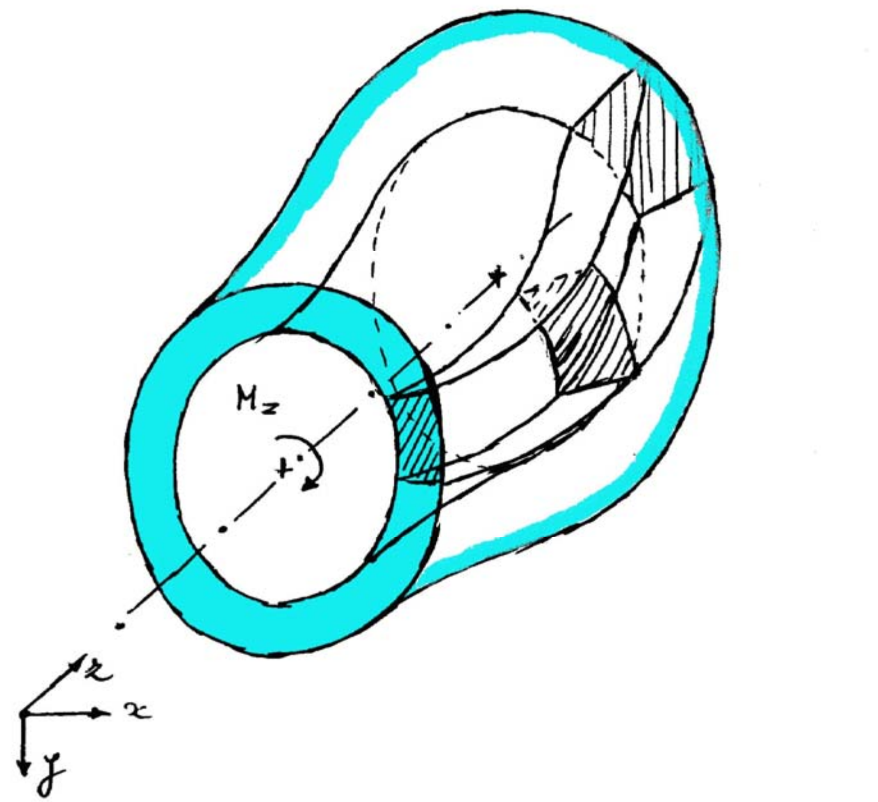
Moto 1D, estensione radiale piccola

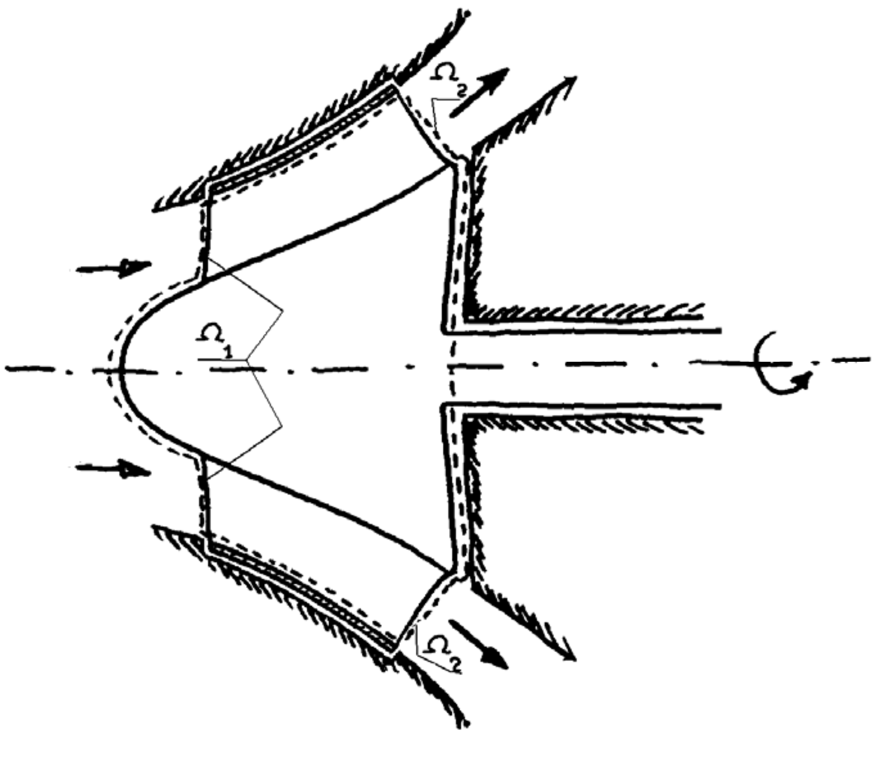
$$\dot{\mathscr{M}}_z = \dot{m}\, r\, c_u$$



Momento forze di volume

$$M_{Vz} = M_{gz} = g\int_V \rho x \, dV$$





$$M_{Vz} = 0$$

Momento delle forze di pressione sulle superfici permeabili

$$M_{\Omega z} = \int_{\Omega_1} \left(\vec{r} \wedge p\vec{n}\right)_z \, d\Omega - \int_{\Omega_2} \left(\vec{r} \wedge p\vec{n}\right)_z \, d\Omega$$

$$\vec{n} = \vec{n}_m$$

$$dM_{\Omega z} = r\,p n_u d\Omega = 0$$

Momento risultante delle forze esercitate dal fluido sulle superfici impermeabili

$$M_{Az} = M_{pz} + M_{\tau z}$$

Momento euleriano

$$\dot{\mathcal{M}}_{ez}\,dt - \dot{\mathcal{M}}_{uz}\,dt + \left[M_{Vz} - M_{Az} + M_{\Omega z}\right]dt = \frac{\partial}{\partial t}\left(\int_V \rho \left|\vec{r}_\perp\right| c_u\, dV\right)dt$$

$$M_\infty = M_{Az} = \dot{\mathcal{M}}_{1z} - \dot{\mathcal{M}}_{2z} + \underbrace{M_{Vz}}_{=0} + \underbrace{M_{\Omega z}}_{=0}$$

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

Momento euleriano

$$M_\infty = \dot{m}\left(r_1 c_{1u} - r_2 c_{2u}\right)$$

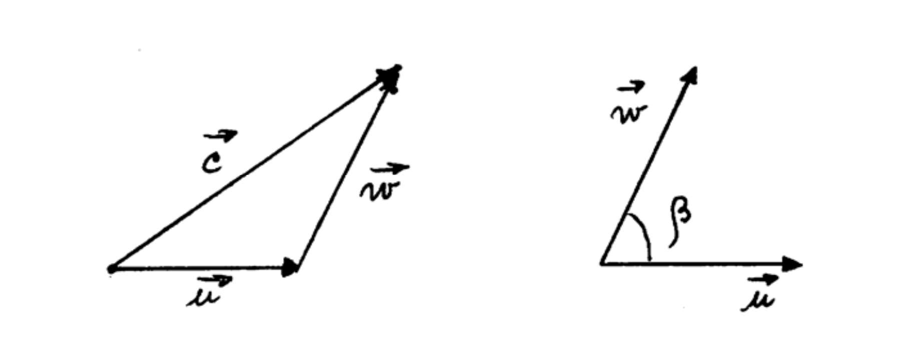
$$\omega M_\infty = P_\infty = \dot{m}\left(\omega r_1 c_{1u} - \omega r_2 c_{2u}\right) = \dot{m}\left(u_1 c_{1u} - u_2 c_{2u}\right)$$

$$L_\infty = \frac{P_\infty}{\dot{m}} = u_1 c_{1u} - u_2 c_{2u}$$

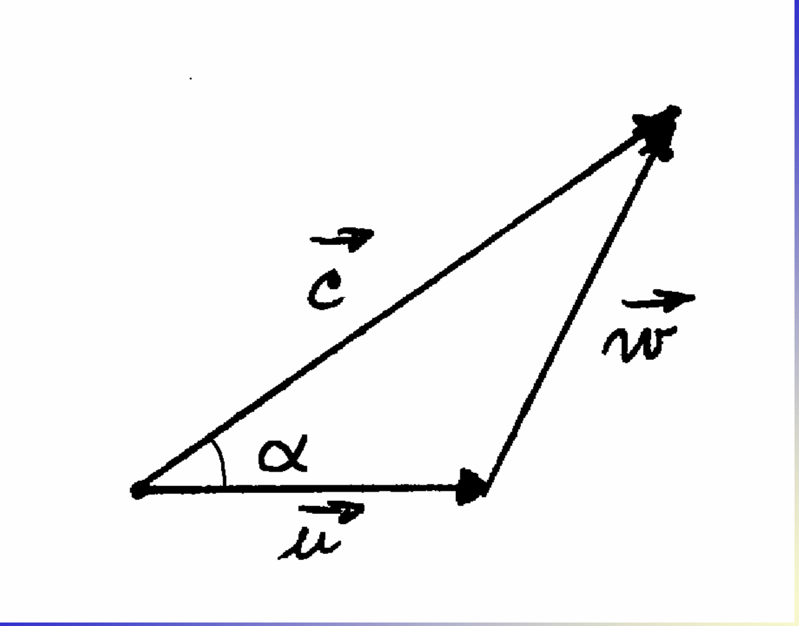
$$L_\infty = u_1 c_1 \cos\alpha_1 - u_2 c_2 \cos\alpha_2$$

Altre forme

$$c\cos\alpha = u + w\cos\beta$$



$$L_\infty = u_1\left(u_1 + w_1\cos\beta_1\right) - u_2\left(u_2 + w_2\cos\beta_2\right) =$$
$$= \left(u_1^2 - u_2^2\right) + \left(u_1 w_1\cos\beta_1 - u_2 w_2\cos\beta_2\right)$$



$$w^2 = u^2 + c^2 - 2uc\cos\alpha$$

$$L_\infty = \frac{1}{2}\left[\left(u_1^2 + c_1^2 - w_1^2\right) - \left(u_2^2 + c_2^2 - w_2^2\right)\right]$$

$$L_\infty = \frac{u_1^2 - u_2^2}{2} + \frac{c_1^2 - c_2^2}{2} + \frac{w_2^2 - w_1^2}{2}$$

Energia specifica rotorica

$$\frac{c_2^2 - c_1^2}{2} + g\left(z_2 - z_1\right) = -L_{12} - \left(h_2 - h_1\right) + q_{e12}$$

$$\frac{c_2^2 - c_1^2}{2} = -L_{12} - \left(h_2 - h_1\right)$$

$$\frac{c_2^2 - c_1^2}{2} = -\left(\frac{u_1^2 - u_2^2}{2} + \frac{c_1^2 - c_2^2}{2} + \frac{w_2^2 - w_1^2}{2}\right) + \left(h_1 - h_2\right)$$

$$h_1 + \frac{w_1^2}{2} - \frac{u_1^2}{2} = h_2 + \frac{w_2^2}{2} - \frac{u_2^2}{2}$$

Entalpia totale
rotorica

$$h_{tR} = h + \frac{w^2}{2} - \frac{u^2}{2}$$

Dall'eq. dell'energia
nel moto relativo

$$wdw - udu = -dh + dq_e$$

$$dh + wdw - udu = 0$$

$$de_p = gdz$$

$$\vec{a}_r = \vec{\omega} \wedge (\vec{\omega} \wedge \vec{r})$$

$$d\vec{F}_r = -\vec{a}_r dm \qquad \vec{u} = \vec{\omega} \wedge \vec{r}$$

$$de_r = d\left(-\omega^2 \int rdr\right) = -\omega^2 rdr = -udu$$

$$wdw - udu = -dh + dq_e$$

$$wdw - udu = -vdp - d\mathcal{L}_a$$

Eq. dell'energia nel moto relativo

Forma meccanica

$$\frac{c_2^2 - c_1^2}{2} + g\left(z_2 - z_1\right) = -\left(\frac{u_1^2 - u_2^2}{2} + \frac{c_1^2 - c_2^2}{2} + \frac{w_2^2 - w_1^2}{2}\right) - \int_1^2 \frac{dp}{\rho} - \mathscr{L}_{a12}$$

$$g\left(z_2 - z_1\right) + \frac{u_1^2 - u_2^2}{2} + \frac{w_2^2 - w_1^2}{2} = -\int_1^2 \frac{dp}{\rho} - \mathscr{L}_{a12}$$

Forma canonica
per l'idraulica

$$z_1 + \frac{w_1^2}{2g} - \frac{u_1^2}{2g} + \frac{p_1}{\rho g} = z_2 + \frac{w_2^2}{2g} - \frac{u_2^2}{2g} + \frac{p_2}{\rho g} + \frac{\mathscr{L}_{a12}}{g}$$

Carico totale rotorico

$$\mathscr{H}_{tR} = z + \frac{w^2 - u^2}{2g} + \frac{p}{\rho g}$$

$$\mathscr{H}_{tR1} = \mathscr{H}_{tR2} + \frac{\mathscr{L}_{a12}}{g}$$